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α spectrum from ^{16}N β decay and the ^{12}C$(\alpha,\gamma)^{16}$O reaction

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New data on the spectrum of α particles following the ^{16}N β decay are used to revise and extend a previous evaluation of the $E1$ part of the astrophysical S factor for the ^{12}C$(\alpha,\gamma)^{16}$O reaction.

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In a previous calculation [1], we used the measured spectrum [2] of α particles from ^{16}N β decay in an attempt to constrain the astrophysical S factor for the ^{12}C$(\alpha,\gamma)^{16}$O reaction. For this test, we used the p-wave part of the total α spectrum that had been separated by Barker [3,4] using an R-matrix parametrization. This test showed the importance of obtaining a better set of data over a much wider energy range, and in particular at lower energies. Here, we update our fit by using a K-matrix parametrization to fit the *total* α spectrum recently obtained by Buchmann *et al.* [5] at TRIUMF.

The K-matrix parametrization for the total α spectrum reads

$$N_\alpha = N_{1\alpha} + N_{3\alpha} \;, \tag{1}$$

with, for $l = 1$ and 3,

$$N_{l\alpha} = f_\beta(E) p_{l\alpha}^2(E) |n_l|^2 / (1 + d_l^2) \;, \tag{2}$$

and

$$n_l = \sum_{\lambda=1}^{3} B_{l\lambda} g_{l\alpha\lambda} / (E_{l\lambda} - E) + b_{l\beta} \;, \tag{3}$$

$$d_l = p_l^2 K_{l\alpha\alpha} \;, \tag{4}$$

$$K_{l\alpha\alpha} = \sum_{\lambda=1}^{3} g_{l\alpha\lambda}^2 / (E_{l\lambda} - E) + b_{l\alpha\alpha} \;. \tag{5}$$

The notation is as in Ref. [1], except that we now explicitly include the l label, rather than assuming implicitly $l = 1$ only. The feeding factors B_{11} and B_{31} for the two bound states (1^- at $E_x = 7.12$ MeV, 3^- at $E_x = 6.13$ MeV) are obtained from Eq. (3.16) of Ref. [1] using the branching ratios from Ref. [6]. The parameter $b_{1\beta}$ is set to zero as in Ref. [1]. The calculated α spectrum has been convolved with the detector resolution for comparison with the data, assuming a Gaussian response whose energy independent full width at half maximum is 35 keV.

As in Ref. [1], we minimize

$$\chi_{\text{eff}}^2 = \frac{1}{3}\left(\chi_{\gamma 1}^2 + \chi_{\delta 1}^2 + \chi_\beta^2\right) \;, \tag{6}$$

where $\chi_{\gamma 1}^2$, $\chi_{\delta 1}^2$, and χ_β^2 are χ^2 per data point for the $E1$ capture cross section, the p-wave elastic scattering phase shift, and the total α spectrum, respectively.

For the f-wave contribution in Eq. (3), we initially fixed the energies E_{31}, E_{32} at -1.032 MeV and 4.415 MeV, respectively, and fixed the energy (E_{33}) of the background pole term. For the remaining parameters, we considered each of the two products $B_{32}g_{3\alpha 2}, B_{33}g_{3\alpha 3}$ as one single free parameter and included $g_{3\alpha 1}$ and $b_{3\beta}$. However, we noted that the detailed form of the f-wave contribution was only weakly constrained by the data. No additional information could be obtained when the f-wave elastic scattering phase shift δ_3 was simultaneously fitted. Apparently, the 3^- resonance at $E_{32} = 4.415$ MeV is too far above the energy range ($E = 1.3 - 1.5$ MeV) that determines the f-wave part of the α spectrum to have a significant effect. After many trials, we concluded that χ_{eff}^2 could not be improved by using more than one free parameter for n_3.

The best result is obtained with a constant n_3,

$$n_3 = b_{3\beta} \;, \tag{7a}$$

but choosing

$$n_3 = B_{31} g_{3\alpha 1} / (E_{31} - E) \tag{7b}$$

gives only a small increase of χ_{eff}^2. The latter form for n_3 is the one adopted in Ref. [5].

The $l = 3$ phase shift also plays a role in determining the denominator of $N_{3\alpha}$ through d_3^2. However, in a *separate* best fit of the δ_3 phase shift, d_3^2 remains much smaller than 1 in the energy range of interest. At $E = 1.4$, 2.4, and 3.2 MeV, d_3^2 is 9.4×10^{-9}, 1.1×10^{-4}, and 1.0×10^{-2}, respectively. Accordingly, we have not varied the parameters of $K_{3\alpha\alpha}$ in d_3 in order to reduce the number of free parameters and better constrain them.

The data we used for the δ_1 phase shift and the $E1$ capture cross section are the same as in Refs. [1,7]. From the TRIUMF data, we removed four data points [8] ($E_\alpha = 1.99 - 2.05$ MeV) at energies close to the 2^+ resonance at $E = 2.68$ MeV. This is important for the α spectrum as there is a known β-decay branching ratio to this state ($\simeq 7 \times 10^{-9}$), and we have not included this state in our parametrization.

The results of two fits corresponding to Eqs. (7a) and (7b), respectively, are given in Tables I and II, while Figs. 1–4 correspond to Eq. (7a) only. The first fit is slightly better than the second one, due to the apparently better parametrization of the f-wave part of the α spec-

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TABLE I. Parameter values for two best fits corresponding to n_3 given by Eq. (7a) (second column) and by Eq. (7b) (third column). As in Ref. [1], $a = 5.46$ fm, and E_{13} is an echo pole. The ranges for S_{E1} correspond approximately to one standard deviation.

E_{11} (MeV)	(-0.0451)	(-0.0451)
$g_{1\alpha 1}a^{-3/2}$ (MeV$^{1/2}$)	-5.408	-5.214
$g_{1\gamma 1}a^{-3/2}$ (MeV$^{1/2}$)	(1.897×10^{-3})	(1.897×10^{-3})
B_{11}	(1193)	(1193)
E_{12} (MeV)	2.4435	2.4437
$g_{1\alpha 2}a^{-3/2}$ (MeV$^{1/2}$)	7.007	7.030
$g_{1\gamma 2}a^{-3/2}$ (MeV$^{1/2}$)	0.6555×10^{-3}	0.6572×10^{-3}
B_{12}	-410.5	-412.8
E_{13} (MeV)	(8.000)	(8.000)
$g_{1\alpha 3}a^{-3/2}$ (MeV$^{1/2}$)	$14.53\,i$	$14.92\,i$
$g_{1\gamma 3}a^{-3/2}$ (MeV$^{1/2}$)	$-2.550\times 10^{-3}\,i$	$-2.641\times 10^{-3}\,i$
B_{13}	$700.5\,i$	$644.9\,i$
$b_{1\alpha\alpha}a^{-3}$	77.72	77.95
$b_{1\alpha\gamma}a^{-3}$	-5.797×10^{-3}	-5.887×10^{-3}
E_{31} (MeV)	(-1.032)	(-1.032)
$g_{3\alpha 1}a^{-7/2}$ (MeV$^{1/2}$)	—	6.780×10^{-2}
B_{31}	—	(2458)
$b_{3\beta}a^{-7}$	-66.75	—
$\Gamma_{1\gamma 1}$ (MeV)	(55×10^{-9})	(55×10^{-9})
$\Gamma_{1\alpha 2}$ (MeV)	0.454	0.457
$\Gamma_{1\gamma 2}$ (MeV)	16×10^{-9}	16×10^{-9}
S_{E1} (MeV b) at $\chi^2_{\min}$	0.045	0.042
S_{E1} (MeV b) range	$0.039-0.050$	$0.035-0.046$

trum with n_3 given by Eq. (7a) rather than by Eq. (7b). The $\chi^2_{\gamma 1}$, $\chi^2_{\delta 1}$ are practically the same for both fits and for our best fit of 1991 [1] using the Wäffler data. Our present results for χ^2_β are significantly larger than those we obtained in Ref. [1]. The uncertainties assigned to the p-wave α data used in Ref. [1] appear to be dominated by a nonstatistical uncertainty in the α particle energy resulting in the very low χ^2_β we obtained previously.

For two different fits to the α spectrum, Ref. [5] has obtained $\chi^2_\beta = 1.46$ and 1.30. While these are somewhat better than ours, Ref. [5] did not simultaneously fit the $l = 1$ phase shift. Not fitting δ_1 has two other consequences. First, the energy of the 1^- resonance is lowered to $E_{12} \approx 2.41$ MeV and the corresponding partial width is reduced to $\Gamma_{1\alpha 2} = 0.371$ MeV. Second, fitting δ_1 data such as those of Plaga *et al.* [9] constrains $g_{1\alpha 1}$ to lower values, explaining our lower S factor compared to Ref. [5]. We do not think one can disregard fitting the presently available data for δ_1, but this problem and the above remarks certainly require further study. More accurate and extended data for δ_1 at low energies would be desirable,

TABLE II. Values of χ^2 for simultaneous fits to the three sets of data and the minimum values of $\chi^2_{\rm eff}$.

	$\chi^2_{\gamma 1}$	$\chi^2_{\delta 1}$	χ^2_β	$\chi^2_{\min}$
In Ref. [1]	0.88	1.51	0.20	0.86
Fit 1	0.87	1.51	1.62	1.33
Fit 2	0.88	1.51	1.78	1.39

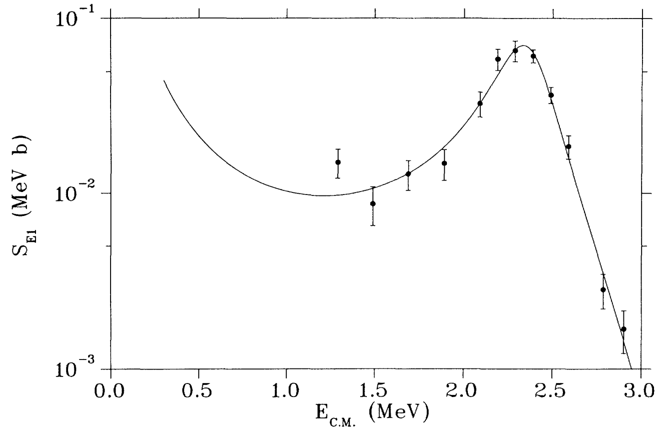

FIG. 1. Fit to the $E1$ S-factor data of Ref. [10] when the δ_1 phase shift and the α spectrum N_α are fitted simultaneously.

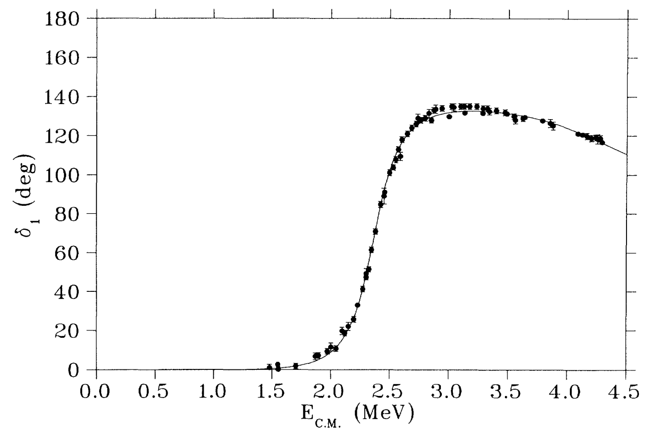

FIG. 2. Fit to the same three sets of phase shift data used in Ref. [1] when the $E1$ S factor and α spectrum N_α are fitted simultaneously.

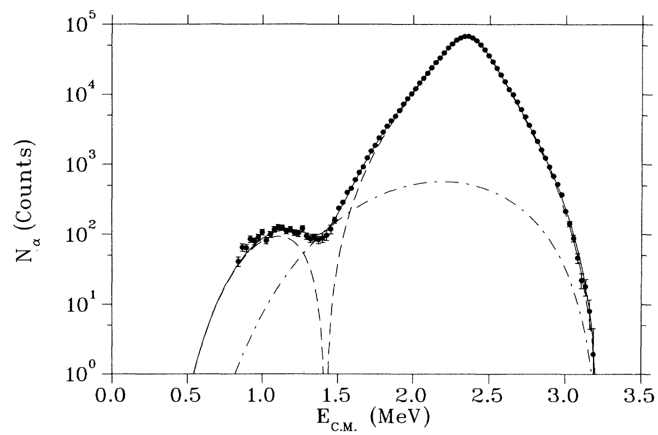

FIG. 3. Fit of the α spectrum to the data of Ref. [5] when the $E1$ S factor and the δ_1 phase shift are simultaneously fitted. The solid curve, the dashed curve, and the dot-dashed curve are the fits of N_α, $N_{1\alpha}$, and $N_{3\alpha}$, respectively.

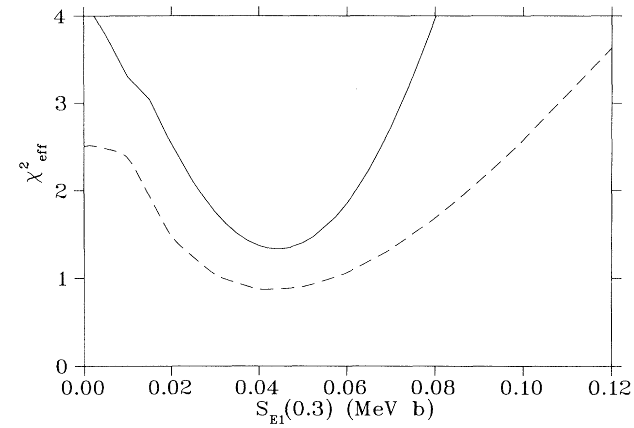

FIG. 4. The effective χ^2 vs $S_{E1}(0.3)$. The minimum value of χ^2_{eff} is obtained for a series of fixed values assigned to $S_{E1}(0.3)$, in the range 0.00–0.12 MeV b. The solid curve corresponds to the first fit in Table I, and the dotted one, shown for comparison, corresponds to the first fit in Ref. [1]. From such curves, we extract the range of allowed values for $S_{E1}(0.3)$.

as well as a better determination of $B_{11} = 1193 \pm 54$. This feeding factor is strongly correlated with $g_{1\alpha 1}$ in $N_{1\alpha}$. It is deduced from the branching ratios given in Ref. [6], and its error is dominated by the branching ratio to the 7.12 MeV bound state.

For both fits reported in Table I, the term involving E_{13} is an echo pole. This implies $g^2_{1\alpha 3} < 0$; i.e., an imaginary $g_{1\alpha 3}$, so that $g_{1\gamma 3}$ and B_{13} must also be imaginary. An echo pole is necessary to parametrize the downward trend of δ_1 at $E > 3.1$ MeV, but no restriction is imposed on the sign of the real products $g_{1\alpha 3}g_{1\gamma 3}, B_{13}g_{1\alpha 3}$. (In an R-matrix fit, a downward trend in the total phase shift is insured by its hard-sphere contribution.)

The χ^2 we have obtained for the capture data, $\chi^2_{\gamma 1} =$ 0.87 and 0.88, are much lower than those reported in Table I of Ref. [5], corresponding to $\chi^2_{\gamma 1} = 2.73$ and 2.69. This is obviously related to the fact that we have fitted $S_{E1}(E)$ only to the data of Kremer *et al.* [10], as in Ref. [1], while in Ref. [5] four sets of independent data are simultaneously fitted.

From the results in Table I and Fig. 4, we obtain a best value for $S_{E1}(0.3)$ of

$$S_{E1}(0.3) = 0.045^{+0.005}_{-0.006} \text{ MeV b} , \tag{8}$$

where the errors correspond approximately to one standard deviation (increase in χ^2_{eff} of 5%, see Ref. [11]). For comparison with Ref. [1], where we quote 95% confidence level errors bars, our errors would be$^{+0.010}_{-0.012}$ (increase in χ^2_{eff} of 20%).

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